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Massive Particles' Tunneling Effect from An Arbitrarily Dimensional Schwarzschild Black Hole

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According to Parikh's recent work, Hawking radiation is viewed as a tunneling process and the barrier is created just by the outgoing particle itself. In this paper, we extend Parikh's work to the case of massive particles' tunneling, and calculate the emission rate at which massive particles tunnel across the event horizon of an arbitrarily dimensional Schwarzschild black hole. The result is also consistent with an underlying unitary theory and takes the same functional form as that of massless particles. Moreover, our result also shows that Hawking radiation is an intrinstic property of the black hole.

KEY WORDS: Hawking radiation; tunneling process; emission rate.

1. INTRODUCTION

Over thirty years ago, Steven Hawking made a striking discovery that a classical black hole could have thermal radiation (Hawking, 1975). So significant it is that it is known, or at least widely agreed, to arise from the combination of quantum mechanics and general relativity. However, there are two crucial questions worthy to be further discussed: information loss and no barrier is found during the tunneling process. Particularly, information loss means that it is not consistent with the underlying unitary theory. In order to overcome these two problems, there are many papers trying to give an interpretation. Recently, Parikh and Wilczek proposed a new method to reconsider the two problems in 2000 (Parikh, 2002, 2004a,b; Parikh and Wilczek, 2000). They treated Hawking radiation as a tunneling process and pointed out that the barrier was created just by the outgoing particle itself. In their papers their key insight is to find a coordinate system well-behaved at the event horizon to calculate the emission rate (Zhang and Zhao, 2005a,b). In this way Parikh and Wilczek obtained the corrected emission spectrum of the particles from many black holes, and the results were all consistent

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with the underlying unitary theory. However, the particles they treated all follow the radial lightlike geodesic when they tunnel across the event horizon, that is, they all are massless. For the massive particles, the radial equation will not be the radial lightlike geodesic. In this paper, we extend Parikh's work to the case of massive particles' tunneling (Zhang and Zhao, 2005c,d) and calculate the emission rate at the event horizon of an arbitrarily dimensional Schwarzschild black hole (Ren *et al.*, 2005). During the calculation, for the sake of simplicity, we consider the outgoing massive particle as a massive shell (de Broglie s-wave). The phase velocity and group velocity of the de Broglie wave corresponding to the outgoing particle are respectively obtained (Zhang and Zhao, 2005c,d), and throughout the study, the units ($G = c = \hbar = 1$) are used.

2. BEHAVIOR OF THE MASSIVE TUNNELING PARTICLES

The line element of an arbitrarily dimensional Schwarzschild black hole is (Ren *et al.*, 2005)

$$ds^2 = -\left(1 - \frac{r_0^n}{r^n}\right) dt_s^2 + \left(1 - \frac{r_0^n}{r^n}\right)^{-1} dr^2 + r^2 d\Omega_{n+1}^2, \tag{1}$$

where

$$r_0^n = \frac{16\pi M}{(n+1)\Omega_{n+1}}. \tag{2}$$

M is the mass of the black hole and Ω_{n+1} is the area of the unit $n+1$ dimensional sphere.

It is easy to find that the metric (1) is singular at the event horizon, $r = r_H = r_0$, As mentioned in Section 1, we first should find a coordinate system which is well-behaved at the event horizon to calculate the emission rate . It's found that the Painleve coordinate system is suitable (Painleve, 1921), and the Painleve line element of the arbitrarily dimensional Schwarzschild is (Ren *et al.*, 2005)

$$\begin{aligned} ds^2 &= -\left(1 - \frac{r_0^n}{r^n}\right) dt^2 + 2\sqrt{\frac{r_0^n}{r^n}} dt dr + dr^2 + r^2 d\Omega_{n+1}^2 \\ &= g_{00} dt^2 + 2 g_{01} dt dr + dr^2 + r^2 d\Omega_{n+1}^2. \end{aligned} \tag{3}$$

here we have made the following transformation

$$t_s = t + f(r), \tag{4}$$

where $f(r)$ satisfies

$$\frac{1}{1 - \frac{r_0^n}{r^n}} - \left(1 - \frac{r_0^n}{r^n}\right) [f'(r)]^2 = 1. \tag{5}$$

The painleve line element (3) displays the stationary, nonstatic, and nonsingular nature of the space time. In particular, it satisfies the following condition (Zhang and Zhao, 2001)

$$\frac{\partial}{\partial x^j}\left(\frac{g_{0i}}{g_{00}}\right) = \frac{\partial}{\partial x^i}\left(\frac{g_{0j}}{g_{00}}\right) \quad (i, j = 1, 2, 3) \tag{6}$$

According to Landu's theory of the coordinate clock synchronization (Landau and Lifshitz, 1975), that is, the coordinate clock synchronization in the Painleve coordinates can be transmitted from one place to another, though the line element is not diagonal. Moreover, it is an instantaneous process in quantum mechanics when particle tunnels across a barrier. Thus, this feature is important for us to discuss the tunneling process.

The radial null geodesics are given by Ren *et al.* (2005)

$$\dot{r} \equiv \frac{dr}{dt} = \pm 1 - \sqrt{\frac{r_0^n}{r^n}} \tag{7}$$

here the upper (lower) sign in (7) corresponding to outgoing (ingoing) geodesics, under the implicit assumption that t increases towards the future.

However, the particles investigated are massive in this paper. Thus, when tunneling across the horizon, they do not follow the radial lightlike geodesics (7). For the sake of simplicity, we consider the outgoing massive particle as a massive shell (de Broglie s-wave). According to the WKB approximation, the wave equation is

$$\Psi(r, t) = Ce^{i\left(\int_{r_i-\varepsilon}^{r} p_r dr - \omega t\right)}. \tag{8}$$

where $r_i - \varepsilon$ represents the initial location of the particle. If we let

$$\int_{r_i-\varepsilon}^{r} p_r dr - \omega t = \phi_0, \tag{9}$$

then, we have

$$\frac{dr}{dt} = \dot{r} = \frac{\omega}{k}. \tag{10}$$

where k is the de Broglie wave number. Comparing (10) with the definition of the phase velocity, we find that $\dot{r}$ is just the phase velocity of the de Broglie wave. To obtain the formula of the phase velocity $\dot{r}$, let us first investigate the behavior of a massive particle tunneling across the horizon.

During the emission, the tunneling across the barrier is an instantaneous process and there are two simultaneous events. One is particle tunneling into the barrier, the other is particle tunneling out of the barrier. In terms of Landau's theory of the coordinate clock synchronization, the difference of coordinate times of these two simultaneous events is

$$dt = -\frac{g_{0i}}{g_{00}}dx^i = -\frac{g_{01}}{g_{00}}dr_c \quad (d\theta = d\varphi = 0), \tag{11}$$

where r_c is the location of the particle. so the group velocity is

$$v_g = \frac{dr_c}{dt} = -\frac{g_{00}}{g_{01}}, \tag{12}$$

For the de Broglie wave, according to the relationship between the phase velocity and the group velocity, the phase velocity is

$$v_p = \dot{r} = \frac{1}{2}v_g = -\frac{1}{2}\frac{g_{00}}{g_{01}}. \tag{13}$$

Substituting g_{00} and g_{01} into (13), we obtain the expression of $\dot{r}$

$$\dot{r} \equiv \frac{dr}{dt} = -\frac{1}{2}\frac{g_{00}}{g_{01}} = \frac{1}{2}\frac{(1 - \frac{r_0^n}{r^n})}{\sqrt{\frac{r_0^n}{r^n}}} = \frac{1}{2}\frac{\left(r^n - r_0^n\right)}{\sqrt{r^n r_0^n}}. \tag{14}$$

which is corresponding to the outgoing motion of the massive particles. Moreover, If the self-gravitation is included (14) should be modified by replacing M with $M - \omega$, where ω is the massive particle's energy.

3. TUNNELING ACROSS THE EVENT HORIZON

For a positive-energy s-wave, the tunneling probability Γ could be written

$$\Gamma \sim \exp(-2\mathrm{Im}\, S) \tag{15}$$

where S is the action, and the imaginary part of the action has been found to have a conveniently simple form (Parikh and Wilczek, 2000)

$$\mathrm{Im}\, S = \mathrm{Im} \int_{r_i}^{r_f} p_r dr = \mathrm{Im} \int_{r_i}^{r_f} \int_0^{p_r} dp'_r dr \tag{16}$$

where p_r is the radial momentum. And $r_i = r_0$ is the initial radius corresponding to the site of pair-creation, which should be slightly inside the event horizon r_H, while $r_f = r'_0$ is the final radius, which is slightly outside the final position of the horizon. We substitute Hamilton's equation $\dot{r} = \frac{dH}{dp_r}\,|_r$ into (16), change the variable from momentum to energy, and switch the order of integration to obtain

$$\mathrm{Im}\, S = \mathrm{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{dH}{\dot{r}} dr = \mathrm{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{2\sqrt{r^n r_0^n} dr}{(r^n - r_0^n)} dM \tag{17}$$

where $H = M$, $M_f = M - \omega$, $M_i = M$. And we fix the total mass and allow the black hole mass to fluctuate. Moreover, we have taken the self-gravitation into consideration.

Do the r integral first in (17), we obtain

$$\operatorname{Im} S = -\frac{2\pi}{n} \operatorname{Im} \int_{M_i}^{M_f} r_0 dM = -\frac{2\pi}{n} \operatorname{Im} \int_{M_i}^{M_f} \sqrt[n]{\frac{16\pi M}{(n+1)\Omega_{n+1}}} dM \tag{18}$$

The imaginary part of the action is

$$\begin{aligned}\operatorname{Im} S &= -\frac{2\pi}{n+1}\left[\frac{16\pi}{(n+1)\Omega_{n+1}}\right]^{\frac{1}{n}}\left[(M-\omega)^{1+\frac{1}{n}} - M^{1+\frac{1}{n}}\right] \\ &= -\frac{1}{8}\Omega_{n+1}\left(r_0^{\prime n+1} - r_0^{n+1}\right)\end{aligned} \tag{19}$$

For an arbitrarily dimensional Schwarzschild black hole, the entropy is (Gao and Shen, 2003; Ren *et al.*, 2005)

$$S = \frac{1}{4} A_H = \frac{1}{4}\Omega_{n+1} r_0^{n+1} \tag{20}$$

Thus, the difference of the entropies of the black hole before and after emission is

$$\begin{aligned}\Delta S_{\mathrm{BH}} &= \Delta S_{\mathrm{BH}}(M-\omega) - \Delta S_{\mathrm{BH}}(M) = \frac{1}{4}\Omega_{n+1}\left(r_0^{\prime n+1} - r_0^{n+1}\right) \\ &= -2 \operatorname{Im} S\end{aligned} \tag{21}$$

and the tunneling rate is therefore

$$\Gamma \sim \exp(-2 \operatorname{Im} S) = e^{\Delta S_{\mathrm{BH}}} \tag{22}$$

which is also consistent with the underlying unitary theory (Ren *et al.*, 2005).

4. DISCUSSION

Although $\dot{r}$ of massive particles in (14) is different from that of massless particles in (7) (selecting the upper sign), it is also consistent with an underlying unitary theory, which shows that the tunneling effect is an intrinsic property of the black hole. Moreover, viewed from the calculation, we find that the results are the same after the r integral is first calculated in (17). They are all equal to

$$\operatorname{Im} S = -\int_{M_i}^{M_f} f(r_H) dM, \quad \text{here } f(r_H) = \frac{2\pi}{n} r_H \tag{23}$$

and the temperature of an arbitrarily dimensional Schwarzschild black hole is (Kanti, 2004)

$$T = \frac{1}{\beta} = \frac{n}{4\pi r_H} \tag{24}$$

Comparing (23) with (24), we can easily obtain

$$f(r_H) = \frac{1}{2}\beta = \frac{1}{2T} \tag{25}$$

which manifests more clearly that the tunneling effect is an intrinsic property of the black hole.

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